Formulas for the two dimensional

Elastic and Chaotic Pendulums

and some 4th order Runge-Kutta methods

Document Version 1.01

Prepared Using LaTeX

by

Stefan Spännare
E-mail: stefan@spaennare.se
November 15, 2007

Contents

1 Introduction

2 The elastic pendulum in two dimensions

3 The chaotic pendulum in two dimensions

4 Runge-Kutta 4th order methods
4.1 First order time derivatives
4.2 Second order time derivatives

5 References

1 Introduction

Note, this document is under development. Please look back for updated versions.

In this document is presented a brief theory (formulas only) for the elastic and chaotic pendulums in two dimensions and some 4th order Runge-Kutta methods with first and second order time derivatives.

The Runge-Kutta methods are given only for the x-coordinate. But the same formulas can be applied to the y-coordinate and z-coordinate (if present) as well.

2 The elastic pendulum in two dimensions

$$\dot{\theta} = \frac{p_\theta}{mr^2}$$

$$\dot{p}_\theta = -mgr\sin(\theta)$$

$$\dot{r} = \frac{p_r}{m}$$

$$\dot{p}_r = \frac{p_\theta^2}{mr^3} - k(r - l_0) + mg\cos(\theta)$$

3 The chaotic pendulum in two dimensions

$$v_1 = -(m_1 + m_2)gl_1\sin(\delta_1) - m_2 l_1 l_2 \dot{\delta}_2^2 \sin(\delta_1 - \delta_2)$$

$$v_2 = -m_2 g l_2 \sin(\delta_2) + m_2 l_1 l_2 \dot{\delta}_1^2 \sin(\delta_1 - \delta_2)$$

$$a_1 = (m_1 + m_2)l_1^2$$

$$a_2 = m_2 l_1 l_2 \cos(\delta_1 - \delta_2)$$

$$a_3 = m_2 l_2^2$$

$$\mathrm{den} = a_1 a_3 - a_2^2$$

$$\ddot{\delta}_1 = \frac{a_3 v_1 - a_2 v_2}{\mathrm{den}}$$

$$\ddot{\delta}_2 = \frac{a_1 v_2 - a_2 v_1}{\mathrm{den}}$$

4 Runge-Kutta 4th order methods

4.1 First order time derivatives

$$h = dt$$

$$A = h \cdot \dot{x}(x_i, t_i)$$

$$B = h \cdot \dot{x}(x_i + \frac{1}{2}A, t_i + \frac{1}{2}h)$$

$$C = h \cdot \dot{x}(x_i + \frac{1}{2}B, t_i + \frac{1}{2}h)$$

$$D = h \cdot \dot{x}(x_i + C, t_i + h)$$

$$x_{i+1} = x_i + \frac{1}{6}(A + 2B + 2C + D)$$

4.2 Second order time derivatives

$$h = dt$$

$$h_2 = \frac{1}{2}h$$

$$A = h_2 \cdot \ddot{x}(x_i, \dot{x}_i)$$

$$\beta = h_2 \cdot (\dot{x}_i + \frac{A}{2})$$

$$B = h_2 \cdot \ddot{x}(x_i + \beta, \dot{x}_i + A)$$

$$C = h_2 \cdot \ddot{x}(x_i + \beta, \dot{x}_i + B)$$

$$\delta = h \cdot (\dot{x}_i + C)$$

$$D = h_2 \cdot \ddot{x}(x_i + \delta, \dot{x}_i + 2C)$$

$$K_1 = \frac{1}{3}(A + B + C)$$

$$K_2 = \frac{1}{3}(A + 2B + 2C + D)$$

$$x_{i+1} = x_i + h \cdot (\dot{x}_i + K_1)$$

$$\dot{x}_{i+1} = \dot{x}_i + K_2$$

5 References

[1] "The Swinging Spring: A Simple Model of Atmospheric Balance" by Peter Lynch, Met Eireann, Dublin, Ireland.

[2] The chaotic pendulum model and the equations for it were found at the University museum at Kulturen in Lund, Sweden.

[3] The 4th order Runge-Kutta with first order time derivatives is found elsewhere on Internet.

[4] The 4th order Runge-Kutta with second order time derivatives was provided by L. Lindegren, Lund Observatory, private communication.